eight under the heading "Algebra"; thirteen under "Theory of numbers"; one hundred and twenty under "Analysis"; twenty-nine under "Groups"; seventy-six under "Geometry"; twelve under "Applied mathematics."]—Volume 21, no. 1, January, 1920: "The strain of a gravitating sphere of variable density and elasticity" by L. M. Hoskins, 1–43; "The geometry of hermitian forms" by J. L. Coolidge, 44–51; "Certain types of involutorial space transformations" by F. R. Sharpe and V. Snyder, 52–78.

AMERICAN DOCTORAL DISSERTATIONS.

E. F. Simonds, "Invariants of differential configurations in the plane," *Transactions of the American Mathematical Society*, 1918, volume 19, pp. 222–250 (Columbia, 1917).

UNDERGRADUATE MATHEMATICS CLUBS.

Edited by U. G. Mitchell, University of Kansas, Lawrence.

CLUB TOPICS.

Although much has already been printed concerning the abacus and its uses,[1] we believe that our readers will find the following article by Professor Leavens decidedly interesting and helpful since it gives an independent discussion based upon the personal impressions and first-hand information of a westerner who has come into contact with the present-day use of the abacus in the far east.

17. The Chinese Suan P'an.

By Dickson H. Leavens, College of Yale in China.

The Chinese suan p'an[2] or abacus is familiar to many from an occasional sight of it on a laundryman's table, but it is perhaps usually regarded either as a device full of the mystery of the East and beyond the grasp of the Occidental, or as an instrument fit only for the ignorant "Celestial" and beneath the notice of one who has studied arithmetic.

A little investigation, however, will show one that it is not only perfectly

[1] Two of the best discussions in English are probably C. G. Knott's article, cited below, and Leslie's *Philosophy of Arithmetic* (Edinburgh, 1820), pp. 15–100. Leslie gives, in great detail, examples of the representation of numbers in different scales of notation and of operations by means of them. From his discussion one can readily see how certain theorems on divisibility of numbers and even the summation of special infinite descending series may be inferred from the use of the abacus.

Some excellent illustrations and references to the literature of the abacus can be found in Smith and Mikami's *History of Japanese Mathematics* cited below. Other readily accessible sources of information are the descriptions given in current histories of mathematics and articles in encyclopedias under the titles "Abacus" and "Calculation."

[2] In this Monthly (1919, 256), it is noted that this word, with various spellings, appears in the *New English Dictionary*, but that "the Chinese word Soroban . . . is not given." Soroban, however, is not Chinese, but Japanese, being the Japanese pronunciation of the same characters, which are used in the written language of both countries.

The same note (following the dictionary) translates suan p'an wrongly as reckoning *board*. The Chinese character for board is romanized pan, hence the error; but it is quite a different character from the one here used, which means a *plate* or *tray;* the pronunciation is also different, p in the most used system of romanization representing practically our b, while p' is similar to our p.

simple, but that it is also not to be despised as an aid in arithmetical work when an adding machine is not available.

The instrument consists simply of a rectangular frame, usually with a back so that it forms a kind of tray, containing a number of rods, about 1/8 inch in diameter, made of wood, bone, or metal, set parallel to each other and to the shorter dimension of the frame. On each rod are strung seven wooden beads, the upper two separated from the lower five by a "bridge" which runs the whole length of the frame. In the Chinese suan p'an, the beads are roughly in the shape of a 60° equatorial zone section of a sphere; in the Japanese soroban, they are in the form of a double cone, with a rather sharp angle where the bases join, and it is claimed that this sharp edge makes for quicker manipulation than the rounded Chinese form.

The most usual size of the instrument is about seven by fourteen inches, and contains thirteen rods or columns, with beads about 7/8 inch in diameter. They may also be obtained with more columns, or with smaller beads, down to about 3/8 inch in diameter; but these are too small for convenience, and the standard size, or a little smaller, is the most practicable. The cost is from fifty cents to a few dollars, depending on the size, and on the quality of the wood.

Each rod represents one column, arranged in decreasing powers of 10 from left to right, as in our system of notation. No decimal point is marked on the ordinary instrument, the user locating it mentally for the problem he is working on. Banks sometimes have the columns labelled, to facilitate computations in the exchange of money. The westerner will find it convenient to tie strings between the columns, separating them into groups of threes; coloring the beads differently in groups of three columns, as is done with the keys of adding machines, might also be a help.

In each column, each of the lower five beads represents 1 unit, and each of the upper two represents 5 units. Properly, only four beads below the bridge, and one above are needed to set up 9 in each column; the Japanese soroban does omit one of the upper beads, thus allowing only 10 to be set up. The Chinese form permits the setting up of 15 in each column; this is never done in addition, as 10 is at once carried to the next column on the left, but in long division it is a convenience to have the extra capacity in setting up the steps of the problem.

It is in this use of one upper bead to represent 5 units that the suan p'an is far superior as a practical device to the abacus of ten beads, often sold as a children's toy or for use in primary arithmetic. On the suan p'an, the eye can take in at a glance the number in a group of beads, never more than four; while on the ten bead form, it is often necessary to stop and count. The arrangement in vertical columns is also a preparation for the decimal notation, much better than the horizontal rows of the American form. Still another advantage is that the beads slide easily on the wooden rods, while in the American form, the thin wires are easily bent, and the beads do not slide so smoothly.

Before beginning a problem, the lower beads are slid down by tilting the instrument, and the upper ones are pushed up with a sweep of the hand, leaving

none adjacent to the bridge. Numbers are set up by moving the beads towards the bridge. The figure below represents the number 15387652 set up, the decimal point to be provided according to the conditions of the problem, and there still being five blank columns to the left.

```
        ---------------------------
        |0 0 0 0 0 0 0 0 0 0 0 0 0|
        |0 0 0 0 0 0   0         0|
        |                         |
        |            0   0 0 0 0  |
Bridge  |-------------------------|
        |          0   0 0 0 0   0|
        |              0 0 0     0|
        |              0 0        |
        |0 0 0 0 0   0         0  |
        |0 0 0 0 0 0 0       0 0  |
        |0 0 0 0 0 0 0     0 0 0 0|
        |0 0 0 0 0 0 0 0 0 0 0 0 0|
        |0 0 0 0 0 0 0 0 0 0 0 0 0|
        ---------------------------
                   1 5 3 8 7 6 5 2
```

The addition of 1 or 2 to this number is done simply by moving up 1 or 2 beads in the unit column. If it is desired to add 3, moving up 3 beads would make 5 beads below the bridge, and we do not wish to use the 5th bead. But adding 3 is the same as adding 5 and subtracting 2, so we bring down one of the upper beads to the bridge, and push down, or as we might say, "erase" the two lower ones. Similarly to add 4, we add 5 and erase 1, etc. To add 8 to 2 (or to any number greater than 2) we add 1 in the next column to the left, and erase 2, etc. Note that we do not actually get 5 beads below, and then change them for 1 above, etc.; we foresee that carrying is necessary and do it as we add. It follows that the same digit may be added in different ways, according to what digit it is to be added to. For instance to add 7 to 0, 1, or 2, we add 5 and 2; to add 7 to 3, 4, 8, or 9, we subtract 3, and add 1 (*i.e.* 10) in the next column to the left; to add 7 to 5, 6 or 7, we add 2, subtract 5, and add 1 (*i.e.* 10) in the next column to the left. Chinese books of instruction give tables of all possible combinations of two digits, showing just when to add the original number, and when to add 5 or 10, and subtract the complementary part. For a child learning to use the instrument, the learning of these tables is perhaps a necessity; but an adult need never see such tables, and with a little practice will soon learn the combinations, so that the fingers make them automatically with little help from the brain.

The addition of numbers of several digits is not done vertically by columns, as in our system, but horizontally by the complete numbers. The first number is set up, then the digits of the second are added, one by one, working from left to right. Then the third number is added, and so on. The working from left

to right is a distinct advantage, as it gives the digits in the order in which they are written or spoken. This enables one person to use the instrument while another calls off the numbers to him, if necessary; although it is of course more often used by one man alone.

For a fuller description of the method of addition, and also for subtraction, multiplication, division, and extraction of roots, the reader is referred to the full and clear explanations given by Knott,[1] and by Smith and Mikami.[2]

Since the principle of the instrument is perfectly simple, it requires only a little practice to develop fair accuracy and speed in the use of it, at least for addition. It is much simpler to learn than typewriting, but as in that, the best way is by regular daily practice. Fifteen minutes daily will in a short time familiarize one with it, so that the carrying becomes practically automatic. This is one of the features of the instrument that recommends it strongly to the writer. There is none of the mental tension of remembering partial totals. If interrupted, you can stop at any point (provided only you keep track of where you leave off) and take it up again. There is of course the objection that if you make a mistake, you must go back to the beginning, whereas in pencil addition, you have some column totals already written down. But even there, you may have no record of the amounts carried, and so have to start afresh. In any addition, the best check is, probably, to add again. The writer has found the abacus very useful in accounting work, which occupies a good deal of his time. As the columns have to have a pencil total anyway, he first adds up with pencil, and then again by abacus. This gives a very good check, as there is almost no chance of making the same mistake by the two different methods.

Another very useful place for the suan p'an is in adding numbers that are not written in columns. The westerner is trained from childhood to add in columns, and it may not be possible for him to become proficient enough on the abacus to save much time there, although as remarked above the work is made easier. But cross addition of large numbers, or the addition of items picked out here and there in an account book, or of numbers on separate slips, as a pile of checks from the bank, is very difficult for the ordinary person, unless he copies them down in a column. It is here that the abacus saves much time, for the left hand can run along from number to number, or turn over the pile of slips, while the right hand adds. The addition should of course be repeated as a check, preferably taking the numbers in the opposite order. There are cases of this kind where a written list is needed, but when only the total is required the abacus is much quicker.

The writer has used the instrument chiefly for addition, and is doubtful whether the abacus methods of multiplication and division have much advantage

---

[1] C. G. Knott, "The abacus in its historic and scientific aspects," *Transactions of the Asiatic Society of Japan*, Yokohama, vol. xiv, 1886. This is reprinted in an abridged form, under the title "The calculating machine of the East: the abacus," in Horsburgh, *Modern Instruments and Methods of Calculation, A Handbook of the Napier Tercentenary*, London, Bell, [1911], pp. 136–154.

[2] D. E. Smith and Y. Mikami, *A History of Japanese Mathematics*, Chicago, Open Court, 1914, Chapter III.

over ours, for one who has not used it from childhood. However, the special case of multiplication where a large number of multipliers are to be used on the same multiplicand, can economically be done on the abacus, by making out a table of the products by the nine digits, and then adding on the instrument the products for each digit of the multiplier, moving one place to the right each time; and if only a certain number of significant figures are wanted, nothing need be added to the right of a certain column. Similar use may be made of the abacus in connection with the various printed partial product tables of interest, etc.

The use of the suan p'an is taught in elementary schools in China by means of a large demonstration instrument, analogous to our demonstration slide rules, which is set on the wall, the beads being prevented from sliding down by bristles set in the rods. Every shopkeeper, and even illiterate coolies, can use it to a certain extent, with great variation in accuracy and speed among individuals, just as there is in written or mental computation with us. Chinese bank clerks are most expert in it, and can go through long calculations of money exchange, which involve two long divisions, such as £ 3,567 5s. 9d. at 6s. $3\frac{1}{2}$d. = Shanghai Taels ? at .731 = Mexican Dollars ?, and give the answer in about the time that it takes us to set down the figures.

While the westerner can probably not hope for such speed, and indeed if he had occasion for it, it would probably be more economical for him to buy an adding machine rather than to spend his time developing it, still the simplicity and cheapness of the instrument might make it a useful help to anyone who has a moderate amount of addition to do, whether in accounting, or in any mathematical work.

CLUB ACTIVITIES.

The Mathematical Club of Smith College, Northampton, Mass. [*1918*, 91, 455].

Officers of the club for the year 1918–19 were: President, Professor Eleanor P. Cushing; vice-president, Barbara Johnson '19; secretary, Emily Porter '19; treasurer, Cornelia Hopkins '19.

November 25, 1918: "Work in the New York Life Insurance Company" by Professor Ruth Wood.

December 2: "Theories of determinants" by Eleanor Smith '19.

December 18: Christmas party.

February 10, 1919: "Theory of determinants" (continued) by Marjorie Stanton '19.

March 3: "Permutations" by Adele Adams '19; "Combinations" by Helen Crittenden '19.

March 24: "Theory of Probability" by Frances Lowe '19 and Gladys Kern '19.

April 28: "Theory of Probability" (continued) by Florence Fessenden '19.

May 19: Social meeting.

The officers elected for the year 1919–20 are: President, Professor Cushing; vice-president, Mary F. McConnaughy '20; secretary, Constance Torrey; treasurer, Ruth Andrew '20.

PI MU EPSILON FRATERNITY, Syracuse University, Syracuse, N. Y. [*1918*, 271–273].

During the year 1918–19 there were forty-one members, of whom sixteen were faculty and graduates and twenty-five were undergraduates. The officers were: Director, Professor John L. Jones; vice-director, Professor Louis Lindsey; secretary, Gertrude Reynolds '19; treasurer, Donald F. Sears '20; librarian, Agnes Wilcox '20; executive committee, the above officers and Roy Horst '19, Helen De Long '19, Ora M. Tanner '19; scholarship committee, Professors Floyd F. Decker and William H. Metzler, Roy Horst '19, Ethel M. Hicks '19 and Lona Preston '19.

December 2, 1918: Outline of plan for the year. Discussion of mathematical magazines.

January 6, 1919: Report of the scholarship committee and election of new members.

January 27: Initiation of new members. "Rerating of regent's papers" by Professor Lindsey.

February 17: "The method of least squares" by Joseph Atwell '19; "An application of the binomial theorem" by William Start '19.

March 10: "Normals to conics" by Ora Tanner '19, Cornelia Tyler '19 and Bertha Adams '19.

March 31: "The planimeter and how to integrate by mechanical means" by Professor Street.

April 28: "Teaching graphs in high school" by Professor Lindsey.

May 12: Election of officers. Informal talks by the faculty and seniors.

May 14: Annual picnic.

May 26: Special meeting to vote on the establishment of a chapter at Ohio State University. (Note: A new chapter of Pi Mu Epsilon has been established at Ohio State University.)

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2822. Proposed by A. M. HARDING, University of Arkansas.

Show that the sum of the series

$$1 + 3\cdot 2 + 5\cdot 2^2 + 7\cdot 2^3 + \cdots + (2n - 1)2^{n-1}$$

to n terms is $3 - 2^n + (n - 1)2^{n+1}$.

2823. Proposed by S. A. COREY, Des Moines, Iowa.

Let TQ and PR be diameters of a circle with center O. Bisect TO at X and draw PQ. On PQ erect the perpendicular XW and on PR, the perpendicular QV. Prove that $OX\cdot PV = PW\cdot PQ$.

2824. Proposed by G. Y. SOSNOW, Newark, N. J.

If n_1, n_2, n_3, n_4 be the lengths of the four normals and t_1, t_2, t_3, the lengths of the three tangents drawn from any point to the semi-cubical parabola, $ay^2 = x^3$, then will $27n_1n_2n_3n_4 = at_1t_2t_3$. [From *Mathematical Tripos Examination*, Cambridge, England.]